Cosmic Rays

19.1 Introduction

Before the development of the ultra high energy particle accelerators, cosmic rays coming from extraterrestrial sources were the only high energy particles available on the earth for the study of elementary particle interactions. The energy for the cosmic ray particles ranges from several tens of MeV to about $10^{20}$ GeV. The upper limit is far beyond anything available in the laboratories on the earth or is likely to be attained in the foreseeable future. Thus interest in cosmic ray research is likely to continue for many more years in the future, though with much lower priority than in the past.

Cosmic rays were discovered by C.T.R. Wilson in England and independently by J. Elster and H. Geitel in Germany in 1900 when they observed that well-insulated charged electroscopes if left as such, gradually lost their charge. It was believed that this was due to some penetrating radiation passing through the electroscopes that ionized the gas in the latter which neutralized the charge on the leaves of the electroscopes.

It was initially thought that the radiation was coming from the traces of radioactive substances present on the earth's crust or in air. However, further research by A. Gockel, V.F. Hess, W. Kolhoerster and others showed that this was not true. The intensity of the radiation was found to change with altitude above the earth's surface. The nature of this intensity variation with altitude is shown graphically in Fig. 19.1. The intensity at first goes down slightly with increasing altitude (i.e., decreasing atmospheric pressure), but then goes up rapidly above about 2000 m which continue up to at least 9000 m. At very high altitude the intensity again decreases slightly with increasing height.

Hess interpreted the above results by assuming that the unknown radiation was mostly coming from beyond the earth. The initial small decrease in the intensity near the sea level showed the diminishing effect of the primary radiation coming from outside. As the primary cosmic rays from extraterrestrial space enter the earth's atmosphere, they are rapidly absorbed due to different kinds of nuclear interactions with the atoms and molecules present in the air. In the process various secondary radiations are produced, the intensity of which gradually builds up with increasing depth below the top layers of the atmosphere. The intensity of the secondary radiation after reaching a maximum begins to diminish due to absorption in air as the atmospheric depth below the top increases. This explains the occurrence of the flat maximum at very high altitude in Fig. 19.1 and the subsequent decrease in the intensity at lower altitudes. The name given to the newly discovered radiation was 'cosmic rays'.

One remarkable property of the cosmic rays is their great penetrability, studied among others by J. Clay, R.A. Millikan, Cameron and others. The presence of the rays was observed at different depths of water in deep lakes as also under deep mines. For example, the intensity of the rays at a depth of 280 m under water was found to be about 1% of the intensity on the surface of the earth. This may be compared with the fact that the intensity of the 2.62 MeV γ-rays from ThC" is about 10% of the intensity on the surface of the earth. This may be compared with the fact that the intensity of the 2.62 MeV γ-rays from ThC" is about 10% of the intensity on the surface of the earth. This may be compared with the fact that the intensity of the 2.62 MeV γ-rays from ThC" is about 10% of the intensity on the surface of the earth.

Fig. 19.2 shows the absorption curve of the cosmic rays in lead which shows the presence of two types of radiation. The absorption curve is determined by using a counter telescope shown in Fig. 19.9 (see § 19.3) with absorbers placed between the two lower counters C₂ and C₃. The telescope is pointed vertically.

One of these, known as the soft component shown by the initial portion of the curve, is more easily absorbed having less penetrability. The other, known as the hard component, is shown by the latter portion of the curve, has much greater penetrability and is very slowly absorbed.
The soft component which is almost completely absorbed by about 10 cm of lead constitutes only 20% of the total intensity at the sea level. The hard component is found to penetrate more than 1 m of lead absorber.

The absorption curve of the cosmic rays in lead:

Since the total penetrating power of the earth's atmosphere is about the same as that of 1 m of lead, the soft component of the cosmic rays observed at the sea level could not have come from beyond the earth. It must have its origin in the earth's atmosphere due to the interaction of the primary cosmic rays with the atoms in the atmospheric air mentioned above. It is made up mostly of high energy electrons, photons and relatively low energy protons and mesons. They constitute secondary radiation.

The hard or penetrating component at sea level is made up of high energy muons (see § 19.8). They are also of secondary origin.

It may be noted that the cosmic rays observed in the earth's atmosphere (except near the top) and at the sea level are mainly the secondary radiation. The primary radiation which, as we shall see later (§ 19.4) consists of very high energy nuclei of different atoms, is mostly absorbed in the uppermost layers of the atmosphere. The absorption curve in Fig. 19.2 shows the absorption of the secondary radiation.  

19.2 Geomagnetic effects of cosmic rays

Worldwide measurements of the cosmic ray intensities have shown systematic variations both with the latitude and longitude (geomagnetic). In addition, azimuthal variation of the cosmic ray intensity has been observed at a particular polar angle about the vertical direction at a particular place, known as the east-west and north-south effects. All these variations can be directly correlated with the effect of the earth's magnetic field on the cosmic ray intensity and are known as geomagnetic effects.

Cosmic Rays:

Latitude effect:

In 1927, the Dutch physicist J. Clay measured the cosmic ray intensity at the sea level at different latitudes while undertaking a sea-voyage between his native country Netherlands and Indonesia (then a Dutch colony) which is located near the geomagnetic equator. He observed that the intensity of the cosmic rays decreased by about 10% from the higher latitudes towards the equator. This is known as the latitude effect.

Clay's initial experiments were followed up by the American physicists A.H. Compton, R.A. Millikan, H.V. Neher and others which confirmed the existence of the latitude effect between about 55° N to 45° S geomagnetic latitudes. The results of these extensive surveys at different altitudes are shown graphically in Fig. 19.3. It is found that at sea level the cosmic ray intensity increases from the geomagnetic equator to about 40° latitude by nearly 14%. For latitudes above 40°, the intensity is almost constant.

Similar measurements at high altitudes have shown that the intensity increases from 0° to 55° latitude above which the intensity is found to be constant. The increase in the intensity at the higher latitudes was found to be much greater than at the sea level.

Explanation of geomagnetic effect:

The latitude effect discussed above shows that the primary cosmic rays coming from extraterrestrial space is mainly made up of electrically charged particles of high energy. Due to the action of the earth's magnetic field they are deflected from their course while coming towards the earth which causes the latitude variation of the cosmic ray intensity.

The Norwegian scientist C. Stoermer developed a theory of the origin of the aurora borealis observed on the polar regions of the earth (1917). According to him the high energy electrons coming from the sun during solar activity strike the atoms and molecules in the upper layers of the atmospheric air and excite the orbital electrons, resulting in the emission of visible radiation which is seen as the aurora. Due to the action of the earth's magnetic field, the electrons coming from the sun
are mostly deflected towards the poles. As such the aurorae are seen in the polar regions only. (The aurora on the south pole is called the aurora australis). Stoermer’s theory was applied by G.E. Lemaitre of Belgium and M.S. Vallarta of Mexico (1933-36) in explaining the latitude effect of the cosmic rays.

The earth’s magnetic field is assumed to be equivalent to that due to a magnetic dipole of moment \( M = 8.1 \times 10^{22} \, \text{J/T} \) located several hundred kilometres from the centre of the earth and pointing roughly south. The field extends to several thousand kilometres above the earth’s surface. The charged particles in the primary cosmic rays while travelling through the long path in this field are deflected by it and are then incident on the uppermost layers of the earth’s atmosphere. The extension of the latter is much less than that of the earth’s magnetic field. As a result the magnetic deflection within the earth’s atmosphere is much less. Stoermer’s theory will be discussed in detail in Appendix A-VI. We shall briefly summarize the main results of the theory here.

The motion of a cosmic ray particle of relativistic mass \( m \) and charge \( q \) coming from a great distance in the vicinity of the earth can be analyzed by solving the differential equation

\[
\frac{d}{dt}(mv) = q(v \times B)
\]

where \( v \) is the velocity of the particle and \( B \) is the magnetic induction due to the earth.

Using spherical polar coordinates \((r, \lambda, \phi)\) as shown in Fig. 19.4, it is possible to obtain two integrals of the motion as given below. Here \( r \) is the distance from the magnetic centre of the earth, \( \lambda \) the geomagnetic latitude and \( \phi \) the geomagnetic longitude increasing from east to west.

If the velocity components are expressed by differentiation w.r.t. \( s = vt \) then one obtains the following two equations:

\[
\begin{align*}
\frac{d}{ds}^2 \cos^2 \lambda \frac{d\phi}{ds} &= \frac{1 - \frac{\cos^2 \lambda}{r^2} - 2\gamma}{r^2} \\
\left(\frac{dr}{ds}\right)^2 + r^2 \left(\frac{d\lambda}{ds}\right)^2 + r^2 \cos^2 \lambda \left(\frac{d\phi}{ds}\right)^2 &= 1
\end{align*}
\]

The second equation is nothing but the conservation of energy. The first gives the \( \phi \)-component of the motion. The motion of the particle in the field of the magnetic dipole of the earth is thus separated into the motion in the meridian plane and motion of the meridian plane about the dipole axis. The meridian plane is taken to be the plane through the position vector of the particle and the geomagnetic axis (z-axis) of the earth’s dipole. The plane has different orientations at different points.

The constant \( \gamma \) in Eq. (19.2-1) is proportional to the angular momentum of the particle parallel to the magnetic moment of the earth at infinite distance.

We shall express the lengths in terms of a unit called the Stoermer unit as defined below: If the radius of the earth is \( R_e \) in metres, then the radius in the Stoermer units is given by

\[
r_s = \frac{R_e}{C}
\]

where

\[
C = \frac{\mu_0 M_e}{4\pi \, m_v}
\]

Obviously the value of \( r_s \) depends on the momentum \( m_v \) of the particle. The charge of the particle is taken to be \( q = e \), the electronic charge. If \( \theta \) is the angle between the trajectory of the particle in space and the meridian plane, then Eq. (19.2-1) transforms to

\[
r_s \cos \lambda \sin \theta + \frac{\cos^2 \lambda}{r_s} = -2\gamma
\]

where \( r_s \) is expressed in Stoermer unit.

Eq. (19.2-5) can be solved for \( \sin \theta \) to give

\[
\sin \theta = -\frac{2\gamma}{r_s \cos \lambda} \frac{\cos \lambda}{r_s^2}
\]

This equation is valid for positive particles. For negative particles, the second terms on the l.h.s. of the Eq. (19.2-5) and on the r.h.s. of Eq. (19.2-6) change sign.

For a given \( \gamma \) only those regions of the meridian plane for which \( |\sin \theta| < 1 \) are available for the particles (allowed regions).

Regions where \( |\sin \theta| > 1 \) are forbidden. Even in allowed regions all points are not available for the cosmic ray particles. Only those regions extending to infinity which are not isolated by forbidden regions are available for the cosmic ray particles.
For $r_s = 1$ the radius of the trajectory of the particle is equal to the earth's radius which happens for a particle of momentum $p_c = 59.6$ GeV. It is found that for $r_s > 1$ the particles can come from all directions, having momenta $p_c > 59.6$ GeV. For $r_s < 1$, $p_c < 59.6$ GeV. These particles can reach the earth from some directions if $\gamma > 1$. This is true for positive particles. If in this case $\gamma < 1$ the particles cannot reach the earth from any direction.

Thus the observable directions of approach of the positive particle to the earth ($r_s = r_e$) are determined by the inequality (with $\gamma > 1$)

$$\sin \theta < \frac{2}{r_e \cos \lambda} \frac{\cos \lambda}{r_s^2}$$

...(19.2-7)

The limiting angle of approach $\theta_C$ is obtained by replacing the inequality in the above equation by an equality sign. $\alpha = \pi/2 - \theta$, is equal to the semivertical angle of a cone known as the Stoermer cone, with the axis along east-west direction perpendicular to the meridian plane such that no particle is able to reach the observation point along directions within this cone (forbidden cone). Fig. 19.5 shows the Stoermer cones for positive (a) and negative (b) particles respectively. Directions such as PA within the Stoermer cone are forbidden while directions PB are allowed. So the positive particles of given $p_c$ can reach a plane on the earth in greater numbers from the west than from the east. For negative particles the converse is true.

\[ \text{Cosmic Rays} \]

It can be shown that the limiting values of the momentum for protons for given $\theta$ and $\lambda$ (with $\gamma > 1$) are given by

$$p_c > 59.6 \left( \frac{1 - \sqrt{1 - \sin \theta \cos \lambda}}{\sin \theta \cos \lambda} \right)^2 \text{ GeV}$$

...(19.2-8)

For $\sin \theta = 0$ (meridian plane), the above inequality reduces to

$$p_c > 14.9 \cos^4 \theta \lambda \text{ GeV}$$

...(19.2-9)

(For details see Appendix A–V)

\[ \text{Motion in the equatorial plane} \]

For better visualization of the allowed and forbidden regions for the cosmic ray particles we consider the motion in the equatorial plane for which $\lambda = 0$. Eq. (19.2-5) for positive particles then reduces to

$$r_s \sin \theta + \frac{1}{r_s} = -2 \gamma$$

...(19.2-10)

The possible paths of the particles for different values of $\lambda$ in the earth's dipole field are shown in Fig. 19.6.

![Fig. 19.5. (a) Stoermer cone for positive particles. (b) Stoermer cone for negative particles.](image-url)

![Fig. 19.6. Possible paths of the cosmic ray particles for different impact parameters (\gamma).](image-url)

For $\gamma = 0$ the impact parameter is zero so that at a great distance from the earth, the particle path is headed for the centre of the earth. The earth's magnetic field pointing up from the plane of the figure deflects the particle to the right (due east). In this case $\sin \theta = -1$ which gives $r_s^2 - 1 = 0$ or $r_s = 1$. All points in the equatorial plane with $r_s > 1$ are allowed. The points within the solid circle $r_s = 1$ are forbidden. Thus if the earth lies within this circle ($r_e < 1$), the particles will not be able to reach...
the surface of the earth. However, if the surface of the earth lies outside the circle \( r_e = 1 \) so that \( r_e > 1 \) all particles reach the earth. The angle \( \theta \) at which the particles are intercepted by the earth can be found from Eq. (19.2-10) by putting \( r_s = r_e \). Since \( r_s \) is a function of the energy, \( \theta \) is also a function of the energy.

We next take the case of \( -2 \gamma = +2.6 \) (i.e., \(-2 \gamma > 2\)) shown at the extreme left of Fig. 19.6. Using Eq. (19.2-10), we then see that a proton of energy 59.6 GeV (radius of the trajectory in the earth’s field equal to \( r_e \)) would come from infinity at \( \theta = 0 \) to reach the point closest to the earth at \( r_s = 2.15 \) for \(-2 \gamma = 2.6\). It will cross the meridian plane from east to west at \( \theta = 90^\circ \) and then continue along its straight line path to \( r = \infty \) with \( \theta \) returning to zero.

For protons of lower energy the situation is more complicated. It turns out that angles of incidence below a limiting value \( \theta_t \) (depending on energy) are forbidden due to the magnetic field. Lower energy corresponds to a circle of smaller radius for the earth (shown by the dashed circle in the figure for \( E = 14 \) GeV). The magnetic field is not strong enough to deflect the particle sufficiently for it to reach the earth’s surface.

As the value of \(-2 \gamma\) decreases to just above \(+2\) the distance of closest approach gets smaller and smaller, the limiting value being \( r_s = 1 \). So the lower energy particle shoots wide of the earth, the surface of which is the dashed circle as above. The minimum energy of the particle must be 59.6 GeV (for a proton) to reach the earth (the solid circle) in this case.

For \(-2 \gamma\) below \(2.0\) the particle will be deflected sufficiently to reach the earth’s surface almost at vertical incidence. For \(-2 \gamma = 1.5\) (say) the particle of lower energy grazes the earth (dashed circle) from west to east. The limiting angle of incidence can be found from Eq. (19.2-7) by putting \( \lambda = 0 \).

**Explanation of latitude effect:**

Lemaitre and Vallarta’s theory discussed above can be used to explain the latitude effect of the cosmic rays.

As can be seen from Table AVI-I in Appendix A-VI, the minimum momenta of the particles reaching a point on the earth from a particular direction (\( \theta \)) becomes smaller with increasing geomagnetic latitude (\( \lambda \)) of the place. This means that as \( \lambda \) increases, more and more particles are able to reach the earth from different directions and hence the intensity goes up with increasing \( \lambda \). As stated before, the earth’s magnetic field acts on the primary cosmic rays. The flattening of the latitude variation curve above about 50° latitude, even at high altitudes, shows that the primary spectrum does not contain particles with the value of the minimum momentum characteristic of \( \lambda = 50^\circ \). Table AVI-I shows that this is around 3 GeV. The difference in the observed variation of the cosmic ray intensity with latitude between sea level and high altitude measurements can be understood as follows. The observation made at sea level are on the secondary cosmic rays produced by the primary rays near the top of the atmosphere. The secondaries produced by the lower energy primaries are mostly absorbed by the atmospheric air while coming down to the sea level. Only the effects of the higher-energy primaries are observed at the sea level. Since their numbers are not much different from the equator to the poles the intensity at the sea level does not change much as we go from the equator to the higher latitudes.

On the other hand, at higher altitudes the secondaries produced by lower energy primaries have appreciable intensities. Since the latter have greater intensities at higher latitudes than at the equator, the latitude variation is much more pronounced at higher altitudes than at the sea level.

Quantitative calculations based on Lemaitre and Vallarta’s theory give the following results. At a particular latitude there is a critical energy \( E_c \) below which the particles cannot reach the earth from the direction \( \theta \) depending on \( \lambda \). For example at the geomagnetic equator (\( \lambda = 0^\circ \)) for \( \theta = 60^\circ \), the critical energy is 36.0 GeV and a cone of semi-vertical angle \( \pi/2 - 60^\circ \) or 30° opening towards the east is excluded for positrons of energy less than this value. For grazing incidence from the east the minimum energy for positrons must be 59.6 GeV. For vertical incidence this is 14.8 GeV. For grazing incidence from the west it should be 10.2 GeV. For protons the energies are slightly different (see Table AVI-I in Appendix A-VI).

![Fig. 19.7. Relative total intensity of the cosmic rays as a function of \( \lambda \) for different \( B_p \).](image)
To calculate the total intensity from all directions use is made of a modification of Liouville's theorem which shows that the intensity is the same in all allowed directions if an isotropic distribution is assumed at infinity. When this intensity is multiplied (for a given energy) by the solid angle subtended by the allowed cone we get the total intensity for the particles of the assumed energy.

Fig. 19.7 shows the plot of the relative total intensity as a function of the geomagnetic latitude for particles of different momenta expressed in terms of the magnetic rigidity \((pc = Bpc)\). The curves can be used to make quantitative predictions about the latitude effect for different magnetic rigidities.

**Longitude effect:**

Cosmic ray intensity at a given latitude and altitude shows variations with the longitude of the place. This is known as the longitude effect. If \(I_{\text{max}}\) and \(I_{\text{min}}\) are the maximum and minimum intensities at two longitudes at a particular latitude measured at a given altitude then the ratio \((I_{\text{max}} - I_{\text{min}})/I_{\text{max}}\) gives a measure of the longitude effect. It amounts to about 5\% at the geomagnetic equator and decreases to zero at higher latitudes. It is found that along the geomagnetic equator, the intensity is the minimum in the Indian ocean.

The longitude effect is due to the eccentricity of the earth's magnetic field caused by the displacement of the earth's geomagnetic field centre. The displacement is about \(\delta = 342\) km from the centre of the earth to a point 6.5° N latitude and 161.8° E longitude. The distance \(R\) from the geomagnetic centre is a function of the longitude \(\phi\) and is given by

\[
R^2 = R_e^2 + \delta^2 - 2R_e\delta\cos(\phi - \phi_0)
\]

where \(R_e\) is the mean radius of the earth assumed spherical and \(\phi_0\) is the geomagnetic longitude of the point on the earth's surface which is nearest to the geomagnetic centre.

The limiting value of the momentum \(pc\) for a given \(\lambda\) calculated from Lemaître-Vallarta theory as discussed above is found to depend on the longitude \(\phi\). Thus for vertical incidence at the equator \((\lambda = 0°)\), it is found that \(pc\) should vary between 13.7 GeV and 16.5 GeV. Theoretical predictions do not agree very well with observed longitude variation. Thus the point of minimum intensity occurs about 65° west of the predicted value close to a point where the earth's field is maximum. Further, the magnitude of the effect requires higher strength of the earth's field than observed at the position of the maximum field strength.

**East-west asymmetry:**

Another geomagnetic effect of considerable significance is the east-west asymmetry of the cosmic ray intensity. In 1933 the American scientists T. H. Johnson working in Mexico city (29° N geomagnetic latitude at an altitude of 2250 m) and L.W. Alvarez working with Compton at the same location found that the intensity of the cosmic rays coming from the west was about 10% higher than that from the east. A few months later S. De Benedetti and B. Rossi of Italy observed 26% higher intensity from the west than from the east at 11° N latitude at an altitude of 2370 m.

The measurements were made using Geiger-Müller counter telescope shown in Fig. 19.9 and described in § 19.3. The telescope is pointed to the east and west of the zenith and the intensities for the same zenith angle for the two directions \(\left(\theta_w = \theta_e\right)\) are compared. If \(I_w\) and \(I_e\) are the two intensities at a particular point of observation then the east-west effect is defined as

\[
\alpha = \frac{I_w}{I_e} - \frac{1}{2}(I_w + I_e)
\]

The magnitude of the east-west effect \((\alpha)\) at a given point of observation depends on the zenith angle \(\theta\) because of the dependence of the lower limit of the energy of the particles able to reach the instruments on the direction of incidence. It also depends on the energy required to penetrate the atmosphere, the effective thickness of which depends on the ray direction. The two effects have opposite dependence on the zenith angle. As a result, the maximum east-west asymmetry occurs at \(\theta = 60°\). The effect is more prominent at higher altitudes and near the geomagnetic equator.

As seen before, for positive cosmic ray particles the Stömer cone (forbidden cone) points to the east so that the cosmic ray intensity coming from the west should be higher than that from the east. On the other hand, if the cosmic ray primaries are negative particles, the intensity from the east should be higher.

**Fig. 19.8. Explanation of east-west effect.**
The observed higher intensity from the west can thus be explained by assuming the cosmic ray primaries to consist of positive particles only. This has been confirmed by direct observations of the primaries at the top of the atmosphere (see § 19.4).

The east-west effect can be explained qualitatively with the help of Fig. 19.8. The figure shows the paths of the cosmic ray particles at different points on the geomagnetic equator. Assuming the magnetic south pole (S) to be located above the plane of the figure it can be seen that if the primary rays are positively charged, they will be deflected to the east by the earth's magnetic field which points upwards from the plane of the figure. To an observer on the earth they will thus appear to come from the west. For negative primaries just the opposite will happen.

19.3 Experimental methods in cosmic ray research

In the early days of cosmic rays investigations electroscopes and ionization chambers were extensively used. Later Geiger-Müller counters, Wilson cloud chambers, nuclear emulsion photographic plates, bubble chambers, spark chambers and scintillation detectors were used for different types of work. The principles of operation of these instruments have been described in detail in Ch. VII. Special techniques have been developed for the use of some of these instruments in cosmic ray research which are described below.

(a) Ionization chambers:

Much of the work on the worldwide survey of the latitude effect of the cosmic rays by Millikan and his associates was carried out by using a spherical ionization chamber of 15 cm diameter with 3 mm thick steel well surrounded during most measurements by 10 cm of lead and a 1/2 inch thick cast iron jacket, equivalent to a total thickness of 11 cm of lead. This arrangement eliminated most of the soft component and only the hard component (muons) was recorded. The apparatus was used with an automatically recording electroscope and the survey was conducted in regions served by commercial ships. Similar apparatus was used by A.H. Compton and R.M. Turner in their latitude effect measurements between Vancouver, British Columbia in Canada and Sydney, Australia.

Earlier ionization chambers were of the integrating uncompensated type (see Ch. VII). In latter day chambers, the average ionization current is usually compensated by some method and only the deviations from the average are measured.

Ionization chambers in which the technique of rapid electron collection is utilized are in increasing use at present. With good amplifiers able to reproduce the pulse shape as a function of time, it is possible to get considerable information about the events causing the total ionization.

(b) Counter telescopes; coincidence and anticoincidence arrangements:

Two or three G-M counters arranged in a line, one above the other as shown in Fig. 19.9 is known as a counter telescope. The G-M counters are usually of the glass-walled type. High energy particles which are present in both the soft and hard components of the cosmic ray secondaries, can penetrate through the glass wall and produce an ionization pulse at the anode.

The counters are connected in coincidence. Such an arrangement permits the detection of only particles which arrive within a small cone about the axis of the telescope as shown in the figure. With three counters as shown in the figure the definition of the direction of arrival is better than with only two counters.

A single high energy particle passes through all the three counters C1, C2, C3 in succession only if they are arranged one above the other in a line as shown in the figure. If the distance between the two extreme counters is large compared to the time of transit of the cosmic ray particle through the three G-M counters then the pulses from the three counters will cut off the currents through the three tubes at the input stage of the coincidence circuit. Thus a positive coincidence output pulse will be obtained which can be amplified and recorded. If the incoming particle passes through the two upper counters C1 and C2 but not through the third (C3), the output pulse at the common plate point of the three input tubes of the coincidence circuit will be too small to be recorded. Thus only when the particle goes through all the three counters C1, C2, C3, it will be detected as a true coincidence.

The recorded coincidence rate should be corrected for the accidental coincidences (see § 7.22).

The counter telescope described above may be directed along different directions to determine the cosmic ray intensity at different zenith angles.

In some experiments it may be necessary to detect the particles passing through the two upper counters C1 and C2 but not through C3. In this case C1 and C2 are connected in coincidence while C3 is connected in anticoincidence. The principle of anticoincidence circuit has been discussed in § 7.22.
The use of anticoincidence counters around a counter telescope helps reduce the rate of spurious counts due to side showers during coincidence counting by the telescope. Such side showers are produced by stray particles incident on some materials present in the vicinity of the telescope.

Instead of G-M counters, scintillation detectors or semiconductor detectors placed in an array may be used to perform coincidence experiments of different types. For example, large plastic scintillators are commonly used in extensive air shower investigations.

In many experiments, the counters are arranged in a geometry different from the one described above (i.e., a counter telescope). This is especially important in the investigation of cosmic ray showers (see § 19.7).

(c) Counter-controlled cloud chamber:

Wilson cloud chambers have played a very significant role in cosmic ray investigations (see § 7.11). The track of a charged cosmic ray particle produced in a cloud chamber can be photographed with suitable illumination and photographic devices. Since there is no knowing when a particle might be passing through the chamber, a very large number of unnecessary and wasteful exposures may be needed to photograph the track of a single particle. To avoid this, P.M.S. Blackett of England introduced the technique of counter-controlled cloud chamber (see Fig. 19.10). The method has been explained in § 7.11.

Another technique connected with the cloud chamber experiments on cosmic rays is to introduce a delay between the production of the coincidence pulse which actuates expansion of the chamber and the photographing of the track. Due to such delay, the ionized droplets along the track of the ionizing particle diffuse out to some extent so that when the track is photographed, the individual droplets become visible. Since the number of such droplets per unit path length of the track is proportional to the specific ionization of the particle, the method permits identification of the particle by comparison with the density of the droplets along the track of some known particle.

In cosmic ray investigations, cloud chambers are usually placed in magnetic fields perpendicular to the chamber. From a measurement of the curvature of the track in magnetic field \( B \) it is possible to determine the momentum of the particle from the relation \( p = Bq \) where \( p \) is the radius of curvature of the track and \( q \) is the charge of the ionizing particle. For very high energy work, superconducting magnets producing intense magnetic fields have been used. Measurements of both the momentum and the specific ionization of the particle can be made with the cloud chamber. Such measurements are necessary for the identification of the particle.

For the precise measurement of the radius of curvature of the tracks, it is necessary to have stereoscopic photographs. Various methods have been developed for this purpose.

Cloud chambers with regularly spaced metal plates within them are used for the measurement of range and energy loss of the particles.

d) Nuclear emulsion technique:

This technique is employed in many cosmic ray investigations. Some outstanding discoveries have been made (such as that of the \( \pi \)-mesons) using this technique.

The use of photographic plates of special types (known as nuclear emulsion plates) with a much larger concentration of the silver bromide and with a much larger thickness of the gelatine than in an ordinary photographic plate for the detection of ionizing particles has been discussed in details in § 7.14.

Thick emulsion plates upto 1000 microns thickness made by Ilford Co. of England have been used in many investigations. For very high energy work, stripped emulsions have been used. A number of these stacked one above the other gives emulsions upto several centimetres thick. For details see § 7.14.

19.4 Nature of primary cosmic rays

The latitude effect of the cosmic rays shows that the primary cosmic rays are charged particles. The east-west effect discussed in § 19.2 shows that these must be positively charged. Since they are able to reach the earth by penetrating through the magnetic field of the latter, it is clear that they must be very energetic. In fact, if they are protons, their minimum energy should be about \( 2.5 \times 10^9 \) eV. If heavier nuclei are present, then their energies must be still higher.

Direct confirmation of the nature of the primary cosmic rays was obtained in 1948 by a group of scientists including P. Freier, E.J. Lofgren, E. Ney, F. Oppenheimer, H.L. Harrod, and B. Peters. They sent some
nuclear emulsion plates and a small counter-controlled cloud chamber to a height of 31,000 metres above the sea level by means of a balloon. The planes of the plates were kept vertical by the action of gravity. The nuclear emulsion plates and the photographs taken with the cloud chamber were developed after they were retrieved. The nuclear emulsion plates showed some very thick tracks (see Fig. 19.11) which showed that they had been produced by high energy and high Z nuclei. From the very narrow δ-ray (secondary electron) tracks coming out of these thick tracks it was possible to determine the charge Z of the particles producing the tracks. It may be noted that the number of δ-rays per unit length of the thick tracks depends on Z.

In later experiments instruments sent by balloons and rockets have improved the data.

The measurements show that the primary cosmic rays are composed of high energy bare nuclei with Z ranging from 1 (proton) to 26 (iron). Nuclei with higher Z were also discovered in later observations (upto Z = 40). The table below gives the approximate distribution of the primaries according to their Z values.

<table>
<thead>
<tr>
<th>Nuclei and the group</th>
<th>Z</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen (H)</td>
<td>1</td>
<td>92.9</td>
</tr>
<tr>
<td>Helium (He)</td>
<td>2</td>
<td>6.3</td>
</tr>
<tr>
<td>Light (L)</td>
<td>3-5</td>
<td>0.14</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>6-9</td>
<td>0.42</td>
</tr>
<tr>
<td>Heavy (H)</td>
<td>≥ 10</td>
<td>0.14</td>
</tr>
<tr>
<td>Virtual (V)</td>
<td>≥ 20</td>
<td>0.04</td>
</tr>
</tbody>
</table>

The intensity of the electrons in the primary is less than 10⁻³ for all λ (geomagnetic latitude) and may even be zero. Unless they come from the sun there can be no neutrons. And of course there can be no muons or mesons because of their rapid decay.

Energy distribution of the primary cosmic rays:

It has been possible to determine the energy distribution of the primary cosmic rays from various experiments. The primary spectrum is found to extend upwards from about 10⁹ eV. Particles below 2.5 × 10⁹ eV cannot reach the earth from any direction.

The lower energy region (10⁹ to 10¹¹ eV) has been investigated by balloon flight measurements of the geomagnetic latitude effect. Nuclear emulsions sent to high altitudes by balloons and rockets have yielded more data in the energy region 10¹¹ to 10¹⁵ eV.

The particle energy is estimated from the characteristic of their nuclear interactions (star production). For energies above 10¹⁵ eV, the estimates come from observations on extensive air showers. Highest energy recorded is in the region 10¹⁹ to 10²⁰ eV which corresponds to about 10 joules. This energy is sufficient to raise a mass of 1 kg through 1 m against the earth's attraction.

The primary spectrum shows a maximum at about 5 × 10⁵ eV below which the intensity falls off rapidly. There is a minimum at about 20 MeV, below which the intensity rises again (see Fig. 19.12a). The figure gives the differential energies between E and E + dE incident per unit area per second per unit solid angle. If this is integrated for all energies above E, we
get the integral energy spectrum. Fig 19.12b shows the integral energy spectrum for the energy region 1 GeV to $10^6$ GeV. The entire spectrum cannot be observed at any point on the earth except at the geomagnetic poles. For other latitudes and for a given direction of incidence only a part of the spectrum is observed due to the cut off by the earth's magnetic field (see § 19.2). The nature of the spectrum is strongly influenced by the solar activity (see later) which reduces the intensity and shifts the minimum to higher energy.

The integral energy spectrum (per nucleon) for the energy region $2 \times 10^3$ eV to $10^{15}$ eV (2 GeV to $10^6$ GeV) can be represented by

$$I(>E) = \int_E^\infty I(E) dE = K E^{-(\gamma - 1)}$$

...(19.4-1)

where $I(E)$ gives the differential intensity and can be written as

$$I(E) = (\gamma - 1) K E^{-\gamma}$$

...(19.4-2)

Here we have neglected the negative sign.

The constants $K$ and $\gamma$ depend on the energy. Table 19.2 gives the values of $K$ and $\gamma$ for the different energy regions.

<table>
<thead>
<tr>
<th>$E$ (GeV)</th>
<th>$\gamma - 1$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10 - 50$</td>
<td>1.7 - 1.8</td>
<td>---</td>
</tr>
<tr>
<td>$200 - 2 \times 10^4$</td>
<td>1.7</td>
<td>$2.1 \times 10^4$</td>
</tr>
<tr>
<td>$15 - 2 \times 10^4$</td>
<td>1.53</td>
<td>---</td>
</tr>
<tr>
<td>$10 - 6 \times 10^4$</td>
<td>1.6</td>
<td>$0.93 \times 10^4$</td>
</tr>
<tr>
<td>$10^2 - 2 \times 10^4$</td>
<td>2.2</td>
<td>---</td>
</tr>
<tr>
<td>$5 \times 10^4 - 10^5$</td>
<td>2.17</td>
<td>$8.7 \times 10^4$</td>
</tr>
<tr>
<td>$10^5 - 10^6$</td>
<td>2.26</td>
<td>$6 \times 10^4$</td>
</tr>
</tbody>
</table>

Though the power laws given above apply to protons, they can be applied to heavier nuclei to give the mean energy per nucleon.

To get the total intensity, the directional intensity given above should be multiplied by $\pi$.

The intensity of the cosmic rays goes down rapidly with energy. $I(E)$ varies over enormous range. At $E = 10^3$ eV, $I \sim 150$ particles per $m^2$ per steradian per second. This falls by a factor of about $10^{18}$ for $E = 10^5$ eV. As an example, a detector with $1 \, m^2$ area will record about 5000 primary particles of energy greater than $10^3$ eV per second per steradian. On the other hand the same detector will record about 1 particle every 2000 years for energy greater about $10^5$ eV.

The average energy per cosmic ray particle comes to be about $10^6$ GeV which is obtained by integrating over the energy spectrum shown in Fig. 19.12a. This gives a value of the average energy density of the cosmic rays to be about 0.3 MeV per $m^2$ near the earth. This value is fairly close to the value of the average densities of the luminous, magnetic and kinetic energies of the interstellar gas. So it must be assumed that the bulk of the cosmic rays is confined within our galaxy by the galactic magnetic field. From the known magnitude of this field, it can be shown that particles with energies up to $10^{16} - 10^{17}$ eV can thus be confined. Still higher energy particles must have originated from outside our galaxy.

One prominent feature of the primary spectrum is the relatively large abundance of the L-group of nuclei (Li, Be, B). If we compare this with the relative abundance of these elements in the universe (see Ch. XIV) there is a marked discrepancy. The abundance of these elements is unusually low in the universe. Their relatively high abundance in the cosmic rays show that their source cannot have been the elements of this group distributed throughout the universe. It is believed that they are mostly produced in the nuclear collisions of the heavier cosmic ray particles with the interstellar gas.

### 19.5 Interaction of the cosmic rays in the earth's atmosphere; Origin of the secondaries

The high energy cosmic ray primaries on entering the earth's atmosphere suffer different types of nuclear interactions in the topmost layers of the atmosphere and produce different kinds of secondary radiation. On the average the primary protons travel through about 700 kg/m$^2$ of the atmosphere before suffering collisions with the atmospheric gas atoms. This depth is about 250 kg/m$^2$ in the case of the $\pi$-particles and less in the case of heavier nuclei. Thus hardly any primary cosmic ray can reach the sea level (atmospheric depth about $10^4$ kg/m$^2$).

When a primary particle collides with a nucleus, the latter breaks up. The incident particle, if it is a complex nucleus, also breaks up. In these processes, many new particles which are mostly hadrons such as nucleons, pions ($\pi^\pm$, $\pi^0$), heavy mesons (K), hyperons and baryon-antibaryon pairs are produced. By far the most important components are the $\pi$-mesons. As we have seen in Ch. XVIII the pion production has a threshold of about 290 MeV in $p$-$p$ collision. At the proton energy of about 1 GeV, about one pion is produced in each collision.

The very high energy imported by the incident primary results in an explosive type of nuclear disintegration in which the nucleus breaks up into many fragments producing a "star". The high energy nucleons thus produced in the upper atmosphere suffer collisions with other nuclei to produce more secondary particles as above.

Many of the pions ($\pi^\pm$) undergo radioactive decay with a mean life $2.6 \times 10^8$ s (in their own rest frames) and produce $\mu^\pm$. Some of the
pions produced with very high energies are absorbed by nuclei to produce nuclear stars. The probability of those events increase at higher energies because the relativistic time dilatation increases their mean life of decay considerably so that their chance of nuclear absorption increases.

Because of their very short mean lives and strong interaction with matter, not many pions can come down to the sea level, though some of them are observed at mountain altitudes. The muons ($\mu$) produced in the decay of the charged pions, because of their relatively longer mean-lives (in their own frames) and weak interaction with matter come down to the sea level and constitute the penetrating or hard component of the cosmic rays. Actually because of relativistic time-dilatation (see § 19.9), the muon lives are considerably lengthened due to which they are able to come down to the sea level without decay.

The neutral pions, because of their very short mean lives ($10^{-16}$ s), decay almost immediately after their production to produce two high energy $\gamma$-ray photons in each disintegration. It is these $\gamma$-rays which are responsible for the initiation of the cascade electronic showers in the air (extensive air showers). As we have stated the electrons, positrons and bremsstrahlung photons thus produced mainly constitute the soft component of the cosmic rays.

Fig. 19.13 shows a schematic diagram of the different types of secondaries in the atmosphere starting from a high energy primary particle near the top of the atmosphere.

We now consider the sequence of events taking place in the atmosphere after a high energy cosmic ray primary (proton) reaches the top of the atmosphere from outer space. Since the primary proton has a mean free path of about 700 kg/m$^2$, it suffers about 10-12 collisions (see above) with the nuclei of the atoms in the atmosphere. In each collision about one-half of its energy is spent in the production of the strongly interacting hadrons. These are emitted in showers, each collision resulting in about 10 particles for a primary proton of $10^3$ GeV energy.

At least 15-20% of the collisions result in the production of pions and kaons. The main part of the energy is spent in these events. In the remaining cases, nucleons and other baryons are produced. Nuclear excitation resulting in the disintegration of the nucleus also occurs.

The pions produced in the cascade produce more particles in hadronic collisions. Thus the charged pions ($\pi^\pm$) with energy above ~ 200 GeV compete with the nucleons to breed more hadrons.

The intensity of the nuclear active component (N) rapidly goes down with the atmospheric depth below the top of the atmosphere due to their loss of energy by ionization and absorption. The neutrons, on the other hand come down to lower altitudes. The intensities of the electronic component ($\gamma$) and the muon component ($\mu$), after reaching maxima at the atmospheric depth of about 2000 kg/m$^2$ go down as shown in Fig. 19.14. The total intensity ($t$) is also shown in the figure.

Fig. 19.13. Production of different kinds of secondary particles by primary cosmic rays.

Some neutrons are observed in the lower layers of the atmosphere. They are produced as secondaries in the nuclear disintegration processes (star production) in the upper layers of the atmosphere. Some of them are able to come down to the lower layers. As we have seen in § 10.15 in Ch. X, some of these neutrons can transform $^14$N nuclei in the air into the long
lived radioactive $^{14}\text{C}$ nuclei ($\tau = 5568$ y). The continual production of these $^{14}\text{C}$ nuclei in the atmosphere and their assimilation by living organisms and plants (by photosynthesis) makes possible the determination of the ages of archeological and anthropological objects by Libby’s method.

Apart from $^{14}\text{C}$, the nuclear reactions induced by nuclear-active components in the upper layers of the atmosphere help maintain equilibrium concentrations of several other nuclear species both radioactive and stable, such as $^{3}\text{H}$, $^{7}\text{Be}$, $^{32}\text{P}$, $^{35}\text{S}$, $^{39}\text{O}$ in the atmosphere as well as on the earth.

### 19.6 Discovery of positrons

Positrons are the antiparticles of the electrons having the same mass, charge (sign positive), spin and the magnetic moment (sign opposite of the latter). Their existence was predicted in 1928 by P.A.M. Dirac of England in his relativistic quantum theory of the electron (see § 6.5).

In 1933 the American scientist C.D. Anderson discovered the positrons in the cosmic rays at the California Institute of Technology. Anderson obtained the photographs of the tracks of some cosmic ray particles produced in a cloud chamber placed in a strong magnetic field of 1.5 T. In a few cases he observed tracks with opposite curvatures of very high energy charged particles. The curvature was due to the action of the magnetic field. The opposite curvatures showed that there were track of both positive and negative particles in the photographs. The ionization density along the tracks showed that the negative particles were definitely electrons. Since the ionization densities of both the positive and negative particles were equal, it seemed that the positive particles had the same mass as the electrons.

However, the tracks with the opposite curvatures could also be produced by negative particles (electrons) travelling in the opposite direction.

To settle this point, Anderson placed a 6 cm thick lead plate across the chamber and obtained the photographs of the tracks of particles going through the plate. In some of the pictures, it was observed that the curvatures of the tracks on the two sides of the plate were different. One such photograph is shown in Fig. 19.15. The difference in the curvatures on the two sides of the plate was due to the loss of energy (and momentum) of the particle in traversing the plate. Since the momentum $p$ of a particle of charge $q$ is related to the magnetic rigidity $Bp$ through the equation, $p = Bpq$, it is evident that the radius of curvature $\rho$ would be smaller after the emergence of the particle from the plate. Thus by noting the change of curvature of the track, it is possible to establish definitely the direction of motion of the particle. Looking at the picture in Fig. 19.15, it is therefore clear that the particle was proceeding from the lower side of the plate upwards, since the radius of curvature is smaller on the upper side. From a knowledge of the direction of the magnetic field it was possible for Anderson to establish definitely that the particle was positively charged.

Anderson proved that the particle could not be a proton. The ionization density along the track showed that the particle carried an electronic charge ($q = e$). It was estimated from the measured values of $Bp$ below and above the plate that the momentum of the particle decreased from 63 MeV/c to 23 MeV/c in going through the plate.

The measured momentum of the particle above the plate (23 MeV/c) gives its kinetic energy from the relation

$$E_k = \sqrt{p^2 c^2 - m_0^2 c^4} - m_0 c^2$$

where $m_0$ is the rest mass of the particle. If it is a proton $m_0 c^2 = 938$ MeV and $E_k = 0.28$ MeV. A proton of such a small kinetic energy should produce a much thicker track than observed and the length of the track should only be about 5 mm. The measured length of the track was greater than 50 cm.

It was thus definitely established by Anderson that the observed particle was a positively charged particle of electronic mass.

Anderson was awarded Nobel prize in physics in 1936 for this very important discovery.

Soon after Anderson’s discovery of the positron, Blackett and Occhialini at Cambridge in England found evidences of the electron-positron pair production by a counter controlled cloud chamber.

### 19.7 Cosmic ray shower

While measuring the cosmic ray intensity with an ionization chamber, sudden short duration increases in the intensity are observed from time to time, which are known as the cosmic ray burst. The phenomena was observed quite early in the investigation of the cosmic rays, both at the sea level and at high altitudes.

In 1931, the Italian physicist Bruno Rossi used a coincidence counting arrangement to show that the sudden temporary increases in the cosmic ray intensity were caused due to the production of new types of particles by the cosmic rays in substances present in the vicinity of the counters.
Rossi first placed three Geiger-Müller counters in a row in the horizontal plane and connected them in coincidence. He observed that from time to time all the three counters were actuated simultaneously to produce a coincidence pulse. Obviously no single particle can actuate all the counters simultaneously, unless it comes from a horizontal direction. On the other hand, if a group of particles is simultaneously produced at a point above the counter tray, then some of these particles passing through the three counters can actuate them simultaneously to produce a coincidence pulse.

Later Rossi arranged the three counters in a triangle, as shown in Fig. 19.16a and placed a lead plate above the top counter C1. Obviously no single particle can actuate all the three counters simultaneously. However, if due to some reason, two or more particles are produced simultaneously in the lead plate, then two of these can actuate the three counters simultaneously, as will be evident from the figure.

![Diagram](image)

**Fig. 19.16.** (a) Discovery of cosmic ray shower; (b) Rossi transition curve.

By changing the thickness of the lead plate, it is observed that the rate of coincidence counting changes, as shown in Fig. 19.16b. The counting rate at first increases with increasing lead thickness and then decreases. It reaches a maximum at a thickness, between 1 to 2 cm of lead. At larger thickness the coincidence counting rate becomes almost constant. The variation of the coincidence rate, as shown in Fig. 19.16b is called the Rossi transition curve.

From the above observations, Rossi concluded that cosmic rays produced a large number of secondary particles in the lead plate. With increasing thickness of the lead plate, the probability of production of the secondary particles increases. When the thickness becomes larger, the secondary particles begin to get absorbed in the lead plate, which explains the nature of variation of the coincidence rate as shown in the figure. The phenomenon is known as cosmic ray shower.

In 1933 P.M.S. Blackett and G. Occhialini found direct evidence of the production of cosmic ray shower in metal plates by means of cloud chamber photographs. An example of such a photograph is shown in Fig. 19.17. Placing the cloud chamber in a magnetic field, it could be shown that both positive and negative particles were present in the shower. The rates of ionization of the shower particles showed that they consisted mainly of electrons and positrons.

![Photo](image)

**Fig. 19.17.** Photograph of cosmic ray shower.

The type of cosmic ray shower described above is known as the electronic or cascade shower.

The Indian scientist H.J. Bhabha and W. Heitler developed a theory of the cosmic ray shower in 1937. At about the same time two American scientists J.F. Carlson and J.R. Oppenheimer independently proposed another theory of shower production. Both the theories gave more or less similar results.

According to Bhabha and Heitler, two different physical processes are responsible for the production of cosmic ray shower, viz., electron-positron pair production and the emission of electromagnetic radiation by charged particles by bremsstrahlung.

It was seen in Ch. VI that when high energy electromagnetic radiation ($E > 2 \text{mc}^2 = 1.02 \text{MeV}$) travelled through matter, it undergoes absorption mainly by electron-positron pair creation. The energy of the photon is equally distributed between the two particles created. On the other hand, a high energy electron (or positron) loses energy while travelling through matter by ionization and radiation. It is well-known that when a high energy charged particle is accelerated or decelerated it emits e.m. radiation, thereby losing a fraction of its energy. This is known as bremsstrahlung (see Ch. V).
Suppose now a high energy photon \( E >> 2m_e c^2 \) produces an electron-positron pair in the nuclear electric field of the atom of the material through which it travels. Both these particles \( (e^- \text{ and } e^+) \) are decelerated due to encounters with the atoms of the medium, which causes them to emit e.m. radiation of high energy by bremsstrahlung. The photons thus created are also of high energy and hence they create electron-positron pairs. Each of the \( e^- \) and \( e^+ \) so created again undergoes bremsstrahlung process and creates high energy e.m. radiation. The photons thus created are also of high energy and hence they create electron-positron pairs. The mathematical theory of the cascade process is quite difficult. We can only present the main results. The theory is developed on the assumption that the angular divergence of the shower particles can be neglected.

(a) Radiative collision of electrons:

The cross section of the emission of a photon with energy between \( E_\gamma \) and \( (E_\gamma + dE_\gamma) \) by an electron of energy \( E \) passing by a nucleus of charge \( Z e \) is given by

\[
\frac{d\sigma_{br}}{dE_\gamma} = 4Z^2 \alpha^2 r_0^2 \frac{dE_\gamma}{E_\gamma^3} \ln \frac{183}{Z^{2/3}} + \text{smaller terms}
\]

...(19.7-1)

where \( \alpha = e^2/4\pi \epsilon_0 h c \) is Sommerfeld's fine structure constant and \( r_0 = e^2/4\pi \epsilon_0 m_e c^2 \) is the classical electron radius.

The quantity within the braces is of the order of unity, so that we can write

\[
d\sigma_{br} = 4Z^2 \alpha^2 r_0^2 \frac{dE_\gamma}{E_\gamma} \ln \frac{183}{Z^{2/3}}
\]

...(19.7-2)

The number of photons with the energy in the interval \( E_\gamma \) to \( (E_\gamma + dE_\gamma) \) which an-electron of energy \( E >> m_e c^2 \) produces in a path length \( dx \) is given by

\[
dn(E_\gamma) = N dx \frac{dE_\gamma}{E_\gamma} \frac{d\sigma_{br}}{dx} \frac{1}{x_0}
\]

where \( x_0 \) known as the radiation unit or cascade unit is given by

\[
x_0 = 4N Z^2 \alpha^2 r_0^2 \ln \frac{183}{Z^{2/3}}
\]

...(19.7-4)

\( N \) is the number of nuclei per unit volume, so that \( N dx \) is the number of nuclei present per unit area in the thickness \( dx \) of the absorber. If the lengths are measured in radiation units we can write \( dx/x_0 = dl \) so that

\[
dn(E_\gamma) = dl \frac{dE_\gamma}{E_\gamma}
\]

...(19.7-5)

For lead, \( x_0 = 0.52 \text{cm}, \) while for air \( x_0 = 330 \text{m}. \)

The radiation length is the distance in which the energy of an electron is reduced to \( 1/e \) of its original value, so that it loses on the average \( (1 - 1/e) \) or 0.632 of its initial energy. The mean energy loss per unit path length is

\[
\frac{dE}{dx} = \frac{E}{x_0}
\]

Thus all energy-losses in a collision are equally probable.

(b) Pair production:

We now consider pair creation by a high energy photon. The cross section of the process in which one of the particles \( (e^- \text{ or } e^+) \) has an energy \( E \) while the other has the energy \( (E_\gamma - E) \) is given by (for complete screening and very high energies)

\[
d\sigma_p = 4Z^2 \alpha^2 r_0^2 \frac{dE_\gamma}{E_\gamma} \int \left[ \frac{1}{3} \left( 1 + \frac{E}{E_\gamma} \right) + \left( \frac{E}{E_\gamma} \right)^2 \right] \ln \frac{183}{Z^{2/3}} + \text{small terms}
\]

...(19.7-7)

Integration over energy gives

\[
\sigma_p = Z^2 \alpha^2 r_0^2 \left( \frac{28}{9} \ln \frac{183}{Z^{2/3}} - \frac{2}{27} \right)
\]

...(19.7-8)

As in the case of \( d\sigma_{br} \) the quantity within the braces in Eq. (19.7-7) is of the order of unity, so that \( d\sigma_p \) assumes the same form as Eq. (19.7-2) for \( d\sigma_{br} \). The primary energy is divided with equal probability between the two particles. The probability that one pair is formed in a path length...
The decrease in the intensity of a beam of photons due to pair production in $dx$ may be written as

$$\frac{dn}{n} = -dx/\lambda_p$$

or,

$$n = n_0 \exp\left(-x/\lambda_p\right)$$

where $\lambda_p$ is the mean free path for pair production given by

$$\frac{1}{\lambda_p} = \frac{28}{9} \left(Z^2 n \alpha Z^2 \right) \ln \frac{183}{Z}$$

so that

$$\lambda_p = \frac{7}{9} x_0$$

(c) Ionization: The size of the cascade shower grows until the energies of the component particles become too small. After this the electrons and positrons begin to lose energy by ionization. The limiting critical energy at which the cascading process stops is equal to the energy lost by the electron in one radiation unit of distance. We call this $E_c$ which is given by

$$E_c = x_0 (dE/dx)_{\text{ion}} = (dE/dl)_{\text{ion}}$$

As seen in Ch. IV the ionization loss per unit path $(dE/dx)_{\text{ion}} \propto Z$ while according to Eq. (19.7-4) $x_0 \propto 1/Z$. Hence the critical energy $E_c \propto 1/Z$.

An approximate expression is given by

$$E_c = \frac{1600 m_0 c^2}{Z}$$

It may be noted that the critical energy given above is of the same order of magnitude (though somewhat less) as the energy $E_c$ at which the rates of radiation loss and ionization loss become equal.

In Table 19.3 the values of $E_c$, the critical energy $E_c$ and the radiation unit $x_0$ are listed for some materials.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Air</th>
<th>Water</th>
<th>Al</th>
<th>Fe</th>
<th>Cu</th>
<th>Pb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_c$ (MeV)</td>
<td>98</td>
<td>111</td>
<td>52</td>
<td>25</td>
<td>22.4</td>
<td>7.0</td>
</tr>
<tr>
<td>$e_c$ (MeV)</td>
<td>120</td>
<td>150</td>
<td>60</td>
<td>30</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>$x_0$ (m)</td>
<td>330</td>
<td>0.43</td>
<td>0.097</td>
<td>0.0182</td>
<td>0.0147</td>
<td>0.0052</td>
</tr>
</tbody>
</table>

Cosmic Rays

Air showers:

The cascade showers discussed above may also be generated in the atmosphere. Several workers in different countries found the existence of such showers, usually known as the air showers, by observing coincidences between two parallel horizontal counters (1938). Such coincidences were observed by P. Auger and others even with the counter distance up to 300 m. These extensive or large air showers are believed to be produced by very high energy particles (electrons, positrons and photons) which then come down towards the sea level with a particle density which may be as high as $\Delta \sim 1000$ per m$^2$.

Theoretical calculations show that the denser showers with $\Delta \sim 10^3$ per m$^2$ are created by particles (electrons) with an energy about $10^{15}$ to $10^{16}$ eV. The less dense showers ($\Delta \sim 10$ per m$^2$) are generated by electrons of energy $\sim 10^{17}$ eV.

The extensive air showers consist mostly of electrons ($e^+$ and $e^-$) and photons. Though some workers, including Auger have suggested that a small fraction of the particles in these showers may contain heavier penetrating particles, no direct experimental evidence has been found in support of this.

The cascade shower theory discussed earlier does not take into account the angular spread of the shower particles and assumes that these particles proceed in the direction of the primary particles, which initiated the shower. However, in considering the extensive air showers, the theory needs modification by taking into account the angular and spatial distribution of the shower particles. The angular spread is believed to be mainly due to Rutherford type elastic scattering of the electrons and positrons. The root mean squared lateral spread $<X(E)>$ in radiation units and the root mean squared angular spread $<\theta(E)>$ are given by

$$<X(E)> = 0.80 E_c / E$$

$$<\theta(E)> = 0.73 E_c / E$$

$E_c$ is a constant of the order of 21 MeV and $E > E_c$. The above expressions find rough agreement with the observed results.

A group of scientists at M.I.T. in the U.S.A. studied the lateral and longitudinal distribution of the shower particles in the extensive air showers, using plastic scintillators. The number of shower particles traversing the different detectors, when plotted against the distance of the detector from the core of the shower, showed a rapid decrease with the distance. The frequency of these showers also shows a rapid decrease with the number of shower particles. Thus the number of showers has been found to decrease from $10^7$ for $1.7 \times 10^6$ particles to only 10 for $2 \times 10^6$ particles.
It is possible to estimate the energy \( E \) (in eV) of the primary proton from the total number \( n \) of shower electrons, using the following empirical relation:

\[
n(E) = AE^{1/x} \quad \ldots \quad (19.7-17)
\]

where \( A = 1.35 \times 10^{-12} \) and \( x = 1.14 \).

**Mechanism of extensive air shower production**

A high energy primary cosmic ray particle, such as a proton or a heavy nucleus, upon entering the atmosphere, gives rise to different types of nuclear events. As a result, high energy protons, neutrons, charged and neutral pions are produced (see § 19.5). Many of the particles thus created (e.g., \( p, \pi^\pm, n \) nuclei) produce further nuclear interactions and give rise to many more such particles. They form a narrow bundle of particles known as the core of the shower, which is usually about a few metres in diameter at the sea level.

The neutral pions decay very fast into two high energy photons (see § 18.3). These photons in the shower core then start cascade electronic showers and give rise to a very large number of electrons and positrons. Some charged pions also decay into muons which thus originate from the shower core.

### 19.8 Discovery of muons

The absorption curve of the cosmic rays in matter shows that at the sea level the cosmic rays consist of a soft component and a hard or penetrating component (see § 19.1). The soft component is more easily absorbed in matter, being almost completely absorbed by about 10 cm thickness of lead absorber. It is mainly made up of the electrons, positrons and photons produced in cascade showers created in air by high energy electrons or photons. A part of it may also consist of low energy heavier particles.

The hard or the penetrating component is made up of a new type of particles, known as muons, which are heavier than the electrons but lighter than the protons (see Ch. XVIII). The identity of the hard component was established in 1938 by C.D. Anderson (the discoverer of positron) and S.H. Neddermeyer. That the penetrating components of the cosmic rays are not high energy electrons, positrons, or photons, can be seen from the fact that they do not initiate any cascade electronic shower. In fact, cloud chamber photographs show that the particles in the hard component are usually single, not being accompanied by shower particles, when they pass through a high Z absorber e.g., lead.

A high energy charged particle loses energy in matter either by ionization or by radiation (see Ch. V). At lower energies, the rate of ionization loss is relatively high and goes down rapidly with increasing energy. After reaching a minimum at an energy equal to about twice the rest mass energy \((2m_e c^2)\) it again rises slowly. On the other hand, the rate of radiation loss (due to bremsstrahlung) becomes important only at very high energies, much above the minimum of the ionization loss curve. In fact it is of importance for the electrons (and positrons) only. For heavier particles it can be altogether neglected. Only the ionization loss is of importance for them. The variations of the specific energy loss due to ionization for electrons and protons with energy are shown in Fig. 5.26 in Ch. V. Also shown in the same diagram is the specific energy loss of electrons due to radiation.

Let us now consider the ionization loss of three type of particles viz. electrons of mass \( m_e \), singly charged particles of mass 200 \( m_e \) and protons (mass 2000 \( m_e \)). The slow rise in the rates of ionization loss after the minima for these particles occur above the energies 2 MeV, 300 MeV and 3 GeV respectively (see Fig. 19.19). The rates of energy loss in this region (minimum loss) are more or less the same for all these three particles. Hence the thicknesses of the tracks of the particles in cloud chamber photographs of all of them should be almost the same, so that it is not possible to distinguish them from one another from the appearances of their tracks near the minimum ionization.

![Fig. 19.19](image_url)  
**Fig. 19.19** Ionization loss of particles of different masses.

As stated, out of the above three types of particles the rates of radiation energy loss can be neglected for the protons and for the particles of mass 200 \( m_e \). It is of importance only for electrons at these high energies, when the ionization loss rate is near the minimum value. So the two heavier particles will be much more penetrating then the electrons at energies above the minima in the ionization energy loss curves.

It we now compare the energy loss rates of a particles of mass 200 \( m_e \), and that of a proton at an energy of about \( 10^6 \) to \( 10^9 \) eV, we may expect the former to be highly penetrating with an ionization density along the track near the minimum. On the other hand, for the proton the penetrability will be much lower and the ionization density along its track will be much greater than the minimum.

Anderson and Neddermeyer found evidence of a particle in the cloud chamber photograph of the cosmic rays which showed minimum
ionization along the track and which was highly penetrating. It was observed that after passing through a lead plate of thickness 3.5 mm placed in the cloud chamber its range was about 4 cm in the chamber gas and the radius of curvature of its track was about 7 cm. Since it did not lose much energy in the lead plate it could not be an electron and must be heavier than the latter. Assuming it to be proton, its energy could be determined from the measured range of 4 cm from which its momentum $p$ and the magnetic rigidity $Bp = \frac{pc}{e}$ could also be found. Since the magnetic field was known ($B = 0.79$ T), the radius of curvature should have been 20 cm and not 7 cm as observed. Thus the particle was not a proton. Since the value of $Bp$ was much less than that for a proton the particle must be lighter than a proton.

So Anderson and Neddermeyer concluded that it was a charged particle which was heavier than an electron but lighter than a proton. Preliminary measurements gave a mass of about 200 $m_e$ for the particle. Both positively and negatively charged particles were observed. The magnitude of their minimum ionization showed that they carried one electronic charge. The name mesotron was suggested for them. Later they were called $\mu$ meson. The currently accepted name is muon ($\mu$).

Soon after Anderson and Neddermeyer's discovery, J.C. Street and E.C. Stevenson working at Harvard University in the U.S.A. carried out an experiment which not only provided definitive support for the discovery but also helped determine the mass of the muon more accurately. As shown in Fig. 19.20, a 10 cm thick lead absorber was placed above a cloud chamber placed in a magnetic field which was triggered by particles going through the three G.M. counters $C_1$, $C_2$ and $C_3$ in coincidence without discharging any of the counters in the lowermost tray ($C_4$). The lead absorber $A$ eliminated the electronic component and only the penetrating component (muons) entered the chamber. Thus those penetrating particles which were energetic enough to traverse $C_1$, $C_2$ and $C_3$ through the lead absorber were recorded by this method. However, if the particles were so energetic that they also traversed the walls of the chamber and the second lead plate within the chamber to emerge below them and enter the counter in $C_4$, they were not recorded. This was made possible by connecting $C_4$ in anti-coincidence with $C_1$, $C_2$ and $C_3$.

Thus the photographs of the tracks of relatively lower energy particles were obtained. These tracks were much thicker than the tracks with minimum ionization as in Anderson and Neddermeyer's experiment.

Street and Stevenson also arranged to operate the chamber such that the charged droplets along the tracks could be counted. Thus the ionization density along the tracks could be determined. In one of the photographs obtained by them, the ionization density was found to be six times the minimum and the magnetic rigidity was $9.6 \times 10^{-2}$ T-m. When these values were compared with those for particles of different masses (singly charged), they were found to fit well with the values for a particle of mass close to $200 m_e$. The principle of the method can be understood by referring to Fig. 19.21 in which the ratio ($I/I_0$) of the ionization density $I$ to the minimum density $I_0$ is plotted as a function of $BR$ for particles of different masses. At relatively lower values of $BR$ the curves are well resolved. Thus for a magnetic rigidity of $10^{-1}$ T-m, the values of $I/I_0$ for two particles of masses $200 m_e$ and $300 m_e$ are 7 and 13 respectively. So at such low values of $BR$, it is possible to establish the identity of the particle without any ambiguity.

Later W.B. Fretter of the University of California, Berkeley in the U.S.A. determined the mass of the muon accurately by an experiment in which two cloud chambers were used, one above the other. By the use of counter control technique, it was so arranged that only when a particle passed through both the chambers, they were both expanded. The upper chamber was placed in a magnetic field, which gave the magnetic rigidity
(and the momentum) of the particle. A number of parallel horizontal lead plates in the lower chamber gave the range of the particle. From these measurements Fretter obtained a fairly accurate estimate of the muon mass.

Later measurements for both $\mu^+$ gave the value $m_\mu = 207.6 e$. The muonic mass has been determined fairly accurately by measuring the energy of the transition $2D_{5/2}$ to $2P_{3/2}$ in a muonic atom. The accepted value of the muon rest mass (in energy units) is

$$m_\mu c^2 = 105.65943 \pm 18 \times 10^{-3} \text{ MeV}$$

or,

$$m_\mu = (206.7686 \pm 0.0005) m_e$$

19.9 Muon decay

Muons are not stable particles. A muon decays by the emission of an electron (or a positron) and two neutrinos (actually one neutrino and one antineutrino). The mean life of decay is about $10^{-6}$ s.

B. Rossi and his associates (1941) used the following method for the measurement of the mean life $\tau_\mu$ of the muons. They measured the intensity of the hard component of the cosmic rays in the vertical direction with the help of a counter telescope at the sea level, as also at different altitudes above the sea level. Even if the muons were stable, there would be a diminution in the intensity from the higher altitudes to the sea level due to the absorption of muons in the mass of air in the two places. If a solid absorber (e.g., graphite) with a thickness equivalent to the air mass between the higher altitude and the sea level is placed above the counter telescope at the higher altitude, then the intensities should be equal at the two places if the muons were stable. However, actual measurements showed that even with such an arrangement, the intensities were not equal at the two places.

The intensity at the sea level was found to be lower than at the higher altitude with the graphite absorber placed above the counter telescope.

Rossi and his associates interpreted this result as being due to the radioactive decay of the muons as they come down from the higher altitude to the sea level. They observed that the vertical intensity was reduced by about 10% with the graphite absorber of 870 kg/m$^2$ thickness placed above the counter telescope at Echo lake, Colorado in the U.S.A. (i.e., at the higher altitude). On the other hand, the diminution in intensity at Chicago (near sea level) was about 20%, the air mass between the two places having an equivalent thickness of 870 kg/m$^2$ of graphite. From the differences in the diminution in the intensities as above they estimated $\tau_\mu \sim 2 \mu$s ($2 \times 10^{-6}$ s).

It may be noted that the measured values of $\tau_\mu$ need relativistic correction, because of the very high velocity with which the muons travel.

It is known from the special theory of relativity that an interval of time measured by a moving observer is increased with respect to that measured by an observer at rest. This is known as time dilation (see Vol. I). Since the observer measuring $\tau_\mu$ in the laboratory is moving relative to an observer sitting in a frame attached to the muon. The value of $\tau_\mu$ will appear longer to the observer in the laboratory. The transformation of a time interval $\tau'$ measured by an observer moving with a velocity $v$ to the interval $\tau$ measured by an observer at rest is given by

$$\tau = \tau' \sqrt{1 - \beta^2}$$

where $\beta = v/c$. Hence the mean life of the muon of velocity $v = \beta c$, as measured by the observer in the laboratory is $\tau'_\mu = \tau_\mu \sqrt{1 - \beta^2}$ where $\tau_\mu$ is the mean life of the muon in the rest frame. Obviously $\tau'_\mu$ will be different for the muons of different velocities. The mean life in the rest frame is thus

$$\tau_\mu = \tau'_\mu \sqrt{1 - \beta^2}$$

It is the value of $\tau_\mu$ which has been quoted above.

Measurement of $\tau_\mu$ by a method similar to that of Rossi and others were made by many workers in different parts of the world. One such measurement was carried out by N.N. Das Gupta and P.C. Bhattacharya of the Calcutta University by measuring the muon intensities at Darjeeling (altitude 2330 m) and at Calcutta (near see level). They found a value of $\tau_\mu \sim 2.5 \mu$s.

In later years, more accurate electronic methods were used for the measurement of $\tau_\mu$. In an experiment performed in 1943, N. Nereson and B. Rossi measured $\tau_\mu$ by a coincidence-anticoincidence device using arrays of G-M counters above and below a brass plate within which low energy cosmic ray muons suffered radioactive decay to produce electrons (or positrons) which entered arrays of G-M counters placed on either side of the brass plate. The higher energy muons which did not decay in the brass plate entered another group of G-M counters placed below the top counters to produce anticoincidence pulses.

The delayed electrical pulses produced by the electrons give rise to coincidence pulses along with the pulses generated by the decaying muons. The amplitudes of these pulses obtained from a time circuit were proportional to the time intervals between the pulses from the muons and the delayed pulses from the decay electrons. In this way the time-distribution of the radioactive decay of the muons in the brass plate was obtained from which $\tau_\mu$ was determined.

A more recent method due to Astbury, Hattersley, Hussain Kemp and Muirhead illustrated in Fig. 19.22 used the $\pi - \mu$ beam from an accelerator which after collimation produced coincidence pulses in the plastic scintillators $S_1$ and $S_2$. The arrival of $\mu$ is signalled by these
(and the momentum) of the particle. A number of parallel horizontal lead plates in the lower chamber gave the range of the particle. From these measurements Fretter obtained a fairly accurate estimate of the muon mass.

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It may be noted that the measured values of \( \tau_\mu \) need relativistic correction, because of the very high velocity with which the muons travel.
coincidence pulses. The subsequent decay of the $\mu$ in the target is detected by the scintillator $S$. The average time interval between the arrival and decay of the muons gives $\tau_\mu$. The currently accepted value is $\tau_\mu = (2.19714 \pm 0.00007) \times 10^{-6}$ s.

\[ \mu_\mu = \frac{e \hbar}{2 m_\mu} \]

This is less than $\mu_e$ by the factor $m_e/m_\mu \sim 1/207$. If however, the muon does not obey Dirac equation, then $\mu_\mu$ should be different from the value given by Eq. (19.10-3). The difference, if any, will reflect the effect of some new type of interaction (other than radiation reaction) involving the muon just as the anomalous magnetic moments of the proton and the neutron from the expected value of a nuclear magneton ($\mu_N = e \hbar /2 M_\mu$) reflects the effect of the interaction involving the virtual emission and reabsorption of the pions in the strong field of the nucleon.

Theoretical calculations of the effect of radiation reaction on the muon magnetic moment gives a value for the muon gyromagnetic ratio $g_\mu = 2 \times 1.0011654$ which would make the muon magnetic moment to be

\[ \mu_\mu = 1.0011654 e \hbar /2 m_\mu \]

Precise determination of $\mu_\mu$ has been made by a method based on the measurement of the quantity $(g_\mu - 2)/2$. The basic idea behind the method is to measure the change in the angle between $s_\mu$ and $p_\mu$ (muon momentum) when the muon passes through a magnetic field $B$. The cyclotron frequency of the muon in this field is

\[ \omega_c = \frac{Be}{2 m_\mu} \]

On the other hand the Larmor frequency of precession of a particle of magnetic moment $P = g_\mu \hbar s$ in this field is given by

\[ \omega_L = g_\mu \frac{Be}{2 m_\mu} \]

If it is a Dirac particle, then $g_\mu = 2$, so that $\omega_c = \omega_L$.

When such a particle travels through a magnetic field, the orientation of $s$ w.r.t. $p$ (momentum in the orbit) remains unchanged. However, for $g_\mu > 2$, $\omega_L > \omega_c$ so that the angle between $s_\mu$ and $p_\mu$ will change continuously, being given by

\[ \phi = (\omega_L - \omega_c) t = \frac{g_\mu - 2}{2} \frac{Be}{m_\mu} \]

Now the time for $n$ cyclotron revolutions is given by $t = 2 \pi n / \omega_c = 2 \pi m_\mu n / Be$. Hence the value of $\phi$ in this time will be

\[ \phi = 2 \pi n \frac{g_\mu - 2}{2} \]

In an experiment performed in 1970 the value of $(g_\mu - 2)/2$ was determined by noting the change in the orientation of $s_\mu$ for a given number of revolutions $n$ in a magnetic field.

As the muons leave the magnetic field they hit a target and decay. The decay positrons produce a signal which gives the time of flight of the
19.11 Interaction of muons with matter

The observed high penetrability of the cosmic ray muons showed that they interact very weakly with matter. This observation goes against the proposition that the muons are the quanta of the strong, short-range internucleonic field visualized by Yukawa. To investigate this point further, a group of Italian scientists performed an experiment described below.

**Experiment of Conversi, Poncini and Piccioni:**

These scientists performed an experiment in which the radioactive disintegration of positive and negative muons in different substances were studied.

They separated the positive and negative muons with the help of a magnetic lens and studied their behaviours in iron (Z = 26) and carbon (Z = 6). They found that almost all the negative muons were stopped in iron while none in carbon. In the latter case they mostly undergo radioactive decay. Such a difference in behaviour is expected because of the Z^4 dependence of the muon capture probability by the nuclei. In the case of positive muons, no such difference in their behaviour in the two absorbers was found.

These observations proved conclusively that there was no strong interaction between the muons and the nuclei of the atoms. If there were any such strong interaction, then the negative muons would be captured in the nuclei of both higher Z (Fe) and lower Z (C) elements within ~ 10^{-18} s and no difference in the behaviour of μ^- in the two absorbers would take place. Since the experiment showed that the μ^- actually rotates in the K-orbit of the μ-atom of C at least for 10^{-6} s, the probability of strong interaction between the muon and the nucleus is less than 10^{-12} of that expected from Yukawa theory.

The above experiment proved conclusively that the muons are different from the Yukawa particles, which being the quanta of the strong internucleonic field, should themselves interact strongly with the nuclei.
where the particle came to rest. The second particle was also found to have a mass of several hundred times the electron mass. Comparison of the grain densities along the two tracks showed that the second particle was somewhat lighter than the first.

According to Powell and his colleagues, the above type of picture shows the decay of a primary or \( \pi \)-meson into a secondary particle. This secondary particle is also heavier than an electron and has been subsequently identified as a muon later by using more sensitive nuclear emulsion plates. Its decay into an electron has also been observed. The mean life of decay of the \( \pi \)-meson was estimated by its discoverers to be between \( 10^{-6} \) to \( 10^{-11} \) s. (Later measurements gave a value of about \( 2.6 \times 10^{-8} \) s). The muon produced in the decay of the \( \pi \)-meson (pion) has a definite range which corresponds to a kinetic energy of about 4.2 MeV.

The pions are not usually observed in the cosmic rays at sea level. It is believed that they are produced in nucleon-nucleon (N-N) collisions in the upper regions of the atmosphere. Because of their much shorter mean life, they disintegrate in flight, producing a muon while coming down through the air. It is these muons which are observed as the hard component of the cosmic rays.

19.13 Origin of cosmic rays

Cosmic rays are found to come to the earth from all directions with the same intensity, which remains practically constant in time. Very small periodic variations associated with the solar day and year and with the seasons have been observed. These will not be considered here.

At first sight it may appear that the nearly uniform directional intensity rules out the possibility of the origin of the cosmic rays at some specific points in the universe. However, more careful considerations show that this may not be true. We know that the earth is surrounded by a magnetic field, which deflects the cosmic rays from their original course so that they may appear to come towards the earth from a direction quite different from their original direction (see discussion on east-west effect in § 19.2). In fact they may even appear to arrive at the earth from a direction opposite to their original direction. So it may not be improbable that the cosmic rays originate from certain specific points in the universe.

We first consider the solar origin of the cosmic rays. It is known that huge masses of ionized gases are emitted from the sun from time to time. This is known as solar activity. When this becomes exceptionally high in proportions, it is called the solar flare. Solar activity produces large distortions in the earth's magnetic field, which is known to affect the transmission of radio signals on the earth. It also affects the intensity of the cosmic rays. The sudden temporary decrease of the cosmic ray intensity due to solar flares is known as Forbush decrease after S.E. Forbush who first observed it in 1937. The change usually occurs about one day after the solar eruption, causing a magnetic storm on the earth. The earth's magnetic field is first suddenly increased by about 0.1% followed by a decrease of a few parts in 1000, which continues for a few hours. The field then gradually returns to the normal value. The cosmic ray intensity also returns to the normal value. The Forbush decreases affects both low energy particles (which can come only at the geomagnetic poles) and the high energy particles.

Because of the sudden changes in the magnetic field near the sun at the time of solar flares, the charged particles (mainly protons) erupting from the sun gain energy by the 'betatron principle'. As seen in Ch. XII, electrons are accelerated in the betatrons due to the changing magnetic flux. The charged particles ejected from the sun gain energies of the order of a few hundred MeV or even up to a few GeV due to the betatron effect. It is believed that at least a part of the cosmic ray intensity observed on the earth is made up of these relatively low energy charged particles from the sun.

However, the entire cosmic ray spectrum that is observed could not have originated from the sun. This conclusion is drawn from the fact that the directional cosmic ray intensity is the same everywhere and remains the same at all hours of the day and night. Considering the very high energy particles (> 1014 eV), it is easy to see that neither the magnetic field of the earth (\( 5 \times 10^{-5} \) T) nor the very feeble magnetic field (~ 10^-9 T) pervading the interplanetary space in the solar system, can bend these particles sufficiently to make them appear isotropic. For a proton of energy 10^{15} eV, the value of the magnetic rigidity is

\[
Bp = \frac{1.6 \times 10^4}{ce} = 3.3 \times 10^6 T \cdot m
\]

In the interplanetary magnetic field, the radius of curvature of such a particle is \( r = 3.3 \times 10^9 / 10^9 = 3.3 \times 10^{15} m \) which is about \( 2.2 \times 10^9 \) times the distance between the sun and the earth (\( 1.5 \times 10^{11} m \)). On the other hand, in the earth's magnetic field, the radius of curvature of such a particle should be about \( 6.6 \times 10^9 \) m which is \( 10^6 \) times, the earth's radius. Thus such high energy particles would appear to come from the direction of the sun, if they were of solar origin. The above estimates rule out the possibility of their solar origin.

Let us now consider the stellar origin of the cosmic rays. If all stars emit the rays equally, then the sun being itself a star, which is nearest to th earth, the cosmic rays on the earth should mainly appear to come from the sun. However, as seen above, this cannot be wholly true except for the relatively low energy particles.

Though the hypothesis that all stars are the sources of cosmic rays cannot be true, the possibility of some special stars being the sources of cosmic rays cannot be ruled out. It is known that some stars in our galactic
neighbourhood undergo gigantic explosions, known as supernova explosions. At the time of supernova explosion, the star’s luminosity increases to thousands of millions time its normal luminosity for a brief period (~ 1 month) and the star explodes in which a large fraction of the mass of the star (nearly 10% of the solar mass) is thrown out with explosive violence. As a result, various types of atomic nuclei are ejected from these stars with very high velocities in different directions. Such supernova explosions occur about once in every 50 years.

The reason for supernova explosion is not fully understood. However, it is believed that nuclear fusion occurring in the interior of star uses up all the primeval hydrogen (protons) to produce helium nuclei, which in turn fuse to produce heavier nuclei till the centre of the star contain only iron nuclei (the most stable of all nuclei). At this point, the gravitational forces are no longer balanced by the radiation pressure, so that gravitational collapse occurs releasing enormous quantity of energy which burns up the hydrogen and other light nuclei in the outer regions of the star and a tremendous explosion (supernova) results. This is responsible for the production of the heavier nuclei. During the supernova explosion, high energy particles are produced which are the main sources of the cosmic rays.

The energy released during the supernova explosion, which is about $10^{44} \text{ J/s}$ is enough to maintain the known energy level of the cosmic rays within our own galaxy (the Milky Way) which is about $10^{49} \text{ J}$. Since the mean life of the cosmic ray particles is about $10^{15} \text{ s}$, the rate of energy loss of these particles in the galaxy is about $(10^{49}/10^{15})$ or $10^{34} \text{ J/s}$ which can thus be replenished by the energy release in the supernova explosion as estimated above. These considerations lead us to believe that one of the main sources of the cosmic rays in our galaxy are the particles ejected during supernova explosions.

There is an alternative hypothesis, according to which the sources of the cosmic rays are the pulsars or rotating neutron stars produced in supernova explosions. The energies of the neutron stars (which are produced with a frequency of about 1 in 50 years) have been estimated to be about $10^{52} \text{ J}$. Assuming only 10% of its rotational energy (~ $10^{42} \text{ J}$) to be emitted as particulate radiation, it can be easily seen that the amount is sufficient to account for the maintenance of the cosmic ray energy level in our galaxy.

There is an important clue which seems to confirm the supernova origin of the cosmic rays. It is known to radio astronomers that the gas clouds left after supernova explosions are strong sources of radio noises. One such well-known source is the Crab nebula which underwent supernova explosion in historically recorded time. The Chinese astronomers observed the supernova explosion of the Crab nebula in 1054 and had kept detailed records of the event. Even now, the gas within the cloud of this nebula seems to be in a state of violent agitation. The reasons for the radio noises associated with the supernova explosion are as follows. The changing electromagnetic field accelerates not only the protons and other heavy nuclei by the betatron principle, but also the electrons. The latter while describing spiral paths around the magnetic field lines, are strong sources of synchrotron radiation, which shows a continuous energy distribution not only in the visible region but also in the radio wave region of the electromagnetic spectrum. Such radiation is not emitted by the heavier particles.

The great intensity of the synchrotron radiation from the Crab nebula indicates the acceleration of a very large number of electrons by betatron principle to quite high energies, probably extending up to several hundred GeV and subsequent radiation loss by them. The same mechanism obviously also accelerates the protons and other heavy nuclei which escape into the interstellar space. Similar processes must be happening in the remains of the other supernovae, which are thus the major sources of the cosmic rays.

**Acceleration mechanism:**

Enrico Fermi suggested the following mechanism for further acceleration of the cosmic ray particles to very high energies in interstellar space. (1951) above the energies with which they were emitted from the supernova.

Astronomical observations have revealed the presence of huge clouds of ionized gas (mostly hydrogen) wandering through interstellar space. They are about 6 to 8 light years in extent. These clouds contain magnetic fields which are stronger than those in the interstellar space between them. The distances between the clouds are of the order of several light years. These magnetized clouds act as magnetic mirrors used for the confinement of plasmas (see Ch. XV). The charged cosmic ray particles are reflected back and forth between two such clouds, just as the plasma particles are reflected back and forth between two regions of high magnetic fields within a magnetic bottle.

If the magnetized clouds were stationary, the particles would not gain or lose any energy on being reflected from them. However, it is known that the magnetized clouds are moving. When reflection takes place from a cloud moving towards a particle it gains energy while reflection from a cloud receding from the particle results in its loss of energy. This can be understood from the following considerations.

Suppose $M_1$ and $M_2$ are two magnetic mirrors coaxial with the $z$-axis in Fig. 19.24. $M_2$ is stationary while $M_1$ is moving along $z$ with the velocity $v_m$. Let a magnetic field $B$ act along $z$ between $M_1$ and $M_2$. The charged particles are reflected back and forth between the two magnetic mirrors.

**Fig. 19.24.** Fermi acceleration mechanism.
the two mirrors. If \( L \) is the distance between \( M_1 \) and \( M_2 \), we can write
\[
v_m = -\frac{dL}{dt}.
\]
Suppose the \( z \)-component of the particle velocity \( v \gg v_m \). During the time taken by the particle to move back and forth between \( M_1 \) and \( M_2 \), the mirror \( M_1 \) moves through the distance \( \Delta L \ll L \).
Let \( v_i \) and \( v_r \) be the incident and reflected velocities of the particle so that the change of velocity on reflection is \( \Delta v = v_r - v_i \) which takes place at intervals of \( 2L/v \).
To an observer sitting on the moving mirror \( M_1 \), the incident and reflected velocities are (see figure)
\[
\bar{v}_i = v_i + v_m, \quad \bar{v}_r = v_r - v_m.
\]
Since \( \bar{v}_i = \bar{v}_r \), we get \( \Delta v = v_r - v_i = 2v_m \).
We then have
\[
\frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{2v_m}{L/v} = \frac{2v_m}{L} - \frac{v}{L} \frac{dL}{dt}
\]
Integration gives \( v_r = \) constant.
Thus as the mirrors move closer together (\( L \) decreases), the velocity \( v \) and the energy of the particle increases. In the reverse case, the particle loses energy.
This is known as the Fermi acceleration mechanism.
It may be noted that since the distances between the magnetized clouds are of the order of several light years, the particles gain energy by this process once every few years. Fermi showed that on an average, there is net gain of energy because head-on collisions occur more frequently.

Another point to note is that for gaining energy by this process, the particles (protons) should have an initial energy in excess of 200 MeV. Most probably they acquire this energy during supernova explosion.
Fermi was able to deduce the energy distribution of the cosmic rays, based on this model which is in broad agreement with the observed distribution.

Cosmic rays from extragalactic region:
So far we have been talking about the origin and acceleration of the cosmic rays within our own galaxy. However, there are reasons to believe that very high energy particles (\( E \sim 10^{18} \) eV or more) probably come from extragalactic region.

Our galaxy (Milky Way) has the shape of a flat disc of diameter about \( 10^5 \) light years (\( \sim 10^{22} \) m). The thickness of the disc is about \( 10^3 \) light years (\( \sim 10^{19} \) m) which is five times as great in the central region. Thus the galactic disc somewhat looks like a solid wheel, the central region resembling the hub of the wheel. Majority of the stars (\( \sim 10^{11} \) in number) are concentrated in the galactic disc, the sun being located in the median plane about two-thirds of the way from the centre. The galactic disc is surrounded by a halo, roughly spherical in shape, having almost the same diameter, in which the density of the stars is much less. The magnetic field within the galactic disc is about \( 5 \times 10^{-6} \) T. Thus the magnetic rigidity of a proton having the radius of the galactic disc is around \( 2.5 \times 10^{11} \) T\( \text{m} \) which corresponds to an energy \( \sim 7.5 \times 10^{19} \) eV.
Because of the flatness of the galactic disc, such a particle cannot be trapped within it by the galactic magnetic field. In fact the particles of such energy cannot even be trapped in the halo region for long.
Thus the particles of energy \( 10^{18} \) eV or more most probably come from beyond our own galaxy.

19.14 Radiation belts

We have seen that cosmic ray particles with energy less than about \( 10^9 \) eV are not observed on the earth due to deflections in the earth's magnetic field. However, in 1958, a type of very high intensity radiation comprising lower energy charged particles was discovered several thousand kilometers above the earth's surface.
The discovery was made by a group of American scientists by sending G-M counters aboard the U.S. satellites Explorer I and Explorer III early 1958. It was observed that an intense radiation belt surrounded the earth in which permanently trapped charged particles travelled back and forth between definite geomagnetic latitudes in the northern and southern hemispheres of the earth. In course of a few years, two such radiation belts were discovered which are known as Van Allen belts after the discoverer J.A. Van Allen. One of the radiation belts is located at an altitude of about 3200 km and the other at the altitude of 16,000 km above the earth. Both the belts are observed at all azimuthal angles about the geomagnetic axis.
The inner belt is mainly made up of protons with energies up to 100 MeV, while the outer belt contains electrons with energies up to a few thousand electron volts and protons of energies up to a few MeV.
The origin of these radiation belts is believed to be due to the action of the earth's magnetic field. We know that the magnetic field lines of the earth extend between the geomagnetic north to the geomagnetic south poles. They are crowded together near the poles, so that the field is stronger near the poles and relatively less so in the equatorial region. Such a distribution of the magnetic field produces the well known magnetic mirror effect discussed in Ch. XV. Charged particles moving in spiralling orbits along the field lines are reflected from the high field region near one of the poles and retrace back their path to be reflected back again from the other pole.
It is known to plasma physicists that for charged particles moving in such a field, the quantity \( W_s/B \) is a constant (adiabatic invariant). Here \( B \) is the magnetic field at any point on the path of the particle and \( W_s = m v^2/2 \) is that part of the kinetic energy of the particle which is due to the component of the velocity of the particle perpendicular
Because of their relatively short half-lives ($\tau = 10.6$ min), they disintegrate soon to produce protons and electrons. It is these protons and electrons which are the main constituents of the inner belt.

On the other hand, the particles in the outer belt are probably of solar origin. The intensity of the radiation in this belt is strongly influenced by the solar activity. It is not fully clear as to whether these particles arrive as such from the sun to be trapped by the earth's field or are first accelerated by bevatron principle in the changing magnetic field of the earth during solar activity and are then trapped in the earth's field.

When a space vehicle passes through the radiation belts, the electrons in the belts produce X-rays of very high intensity which are harmful to the men and materials within the vehicle. To protect from this, proper shielding must be provided.

The electrons in the outer belt are often scattered by the air molecules in the upper layer of the atmosphere to come down to lower altitudes. This may also be caused due to the distortion of the earth's magnetic field by solar activity. The excitation and ionization of the atoms in the upper regions of the atmosphere by these electrons cause emission of visible radiation seen as the aurorae in the polar regions of the earth.

References


Problems

1. The radius of the solar system is \(1.2 \times 10^{13}\) m and the magnetic induction within it is \(10^{-6}\) T. What is the maximum energy of the protons which can be confined within the solar system?

(3600 GeV)

2. Calculate the kinetic energy of the muon emitted in the decay of a pion at rest assuming \(m^p = 139.58\) MeV and \(m_\mu = 105.66\) MeV.

(4.2 MeV)

3. Calculate the maximum kinetic energy of the electrons in the decay of a \(\mu^-\).

(53 MeV)

4. A positron of kinetic energy \(4m_0c^2\) collides with an electron at rest. Two photons are emitted at equal angles \(\theta\) w.r.t. to the direction of the incident positron. Find \(\theta\).

(35.3°)

5. Prove that in Anderson’s experiment on the discovery of positrons if the particle emerging from the lead plate is a proton (of momentum \(23\) MeV/c), then its kinetic energy should be \(0.3\) MeV. What will be its kinetic energy if it is a positron?

(22.5 MeV)

6. Calculate the radius of the earth in Steörmer unit for protons of kinetic energies 1, 10, 59.6 and 100 GeV. The magnetic dipole moment of the earth is \(M = 8.1 \times 10^{22}\) J/T. Radius of the earth = 6378.16 km.

7. Show that the radiation unit of length in the electronic cascade shower theory has the values 0.52 cm and 330 m respectively in air and lead.

8. What is the percentage change in the mean life of muons of total energy 500 MeV?

APPENDIX A-I

ELECTRIC QUADRUPOLE MOMENT

The potential at a point \(P(r)\) due to an arbitrary charge distribution of density \(\rho(r')\) is given by

\[
\Phi (r) = \frac{1}{4\pi \varepsilon_0} \int \frac{\rho(r') \, dr'}{r} = \frac{1}{4\pi \varepsilon_0} \int \frac{\rho(r) \, dr'}{r_1} \quad \text{(AI-1)}
\]

where \(r_1 = |r - r'|\) is the distance of the field point \(P\) from the element of volume \(dr'\) in the source region (see Fig. A-1). We can expand the function \(f(r_1) = 1/r_1\) as follows:

\[
f(r_1) = \frac{1}{r_1} = f(r) + \sum \frac{\partial f}{\partial x_i}(1) + \frac{1}{2} \sum \frac{\partial^2 f}{\partial x_i \partial x_j}(1) + \cdots
\]

\[
= \frac{1}{r} - \sum \frac{\partial}{\partial x_i} \left( \frac{1}{r} \right) + \frac{1}{2} \sum \frac{\partial^2}{\partial x_i \partial x_j} \left( \frac{1}{r} \right) + \cdots \quad \text{(AI-2)}
\]

where we have used the relation \(\frac{\partial}{\partial x_i} = -\frac{\partial}{\partial x_i} x_i\). Here \(i, j\) can have the values 1, 2 and 3 each corresponding to \(x_1 = x, x_2 = y, x_3 = z\). We then have

\[
\Phi = \frac{1}{4\pi \varepsilon_0} \int \rho(r') \left[ \frac{1}{r} - \sum \frac{\partial}{\partial x_i} \left( \frac{1}{r} \right) + \frac{1}{2} \sum \frac{\partial^2}{\partial x_i \partial x_j} \left( \frac{1}{r} \right) + \cdots \right] \, dr'
\]

\[
\quad \text{(AI-3)}
\]